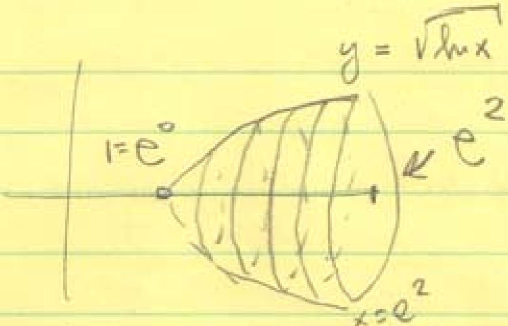
1)



$$V = \int \pi r^2 dx = \pi \int_{x=1}^{x=e^2} [f(x)]^2 dx = \pi \int_{x=1}^{x=e^2} \ln x \, dx$$

$$\int \underbrace{\ln x}_{U} \underbrace{dx}_{dV} = UV - \int V dU = x \ln x - \int \frac{\cancel{x} \, dx}{\cancel{x}} = x \ln x - x = x(\ln x - 1)$$

$$\begin{cases} dU = \dfrac{dx}{x} \\ V = x \end{cases}$$

$$\int_{x=1}^{x=e^2} \ln x \, dx = \Big[x(\ln x - 1) \Big]_{x=1}^{x=e^2} = F(b) - F(a)$$

$$F(b) = e^2 \cdot (2-1) \quad ; \quad F(a) = 1(0-1) = -1$$

$$F(b) - F(a) = e^2 - 1$$

$$V = \pi (e^2 - 1)$$

2) $W = \int_{s_1}^{s_2} dW$ $\quad \boxed{P = \frac{nRT}{V}}$

$dW = F\,ds = \cancel{P}A\,ds = \frac{nRT}{\cancel{V}}A\,ds$

$\boxed{P = \frac{F}{A}} \Rightarrow F = P \cdot A \qquad V = s \cdot A$

$dW = \frac{nRT}{\cancel{V}} A\,ds = \frac{nRT}{s \cdot \cancel{A}} \cancel{A}\,ds$

$$dW = \frac{nRT}{s}\,ds$$

$$W = \int dW = \int \frac{nRT}{s}\,ds = nRT \int_{s_1}^{s_2} \frac{ds}{s}$$

$$W = nRT \left[\ln s\right]_{s_1}^{s_2}$$

$$= nRT\left[\ln s_2 - \ln s_1\right]$$

$$= nRT \ln\left(\frac{s_2}{s_1}\right)$$

3)

$$\int \cos^2 x\,(3-\sin x)\cos x\,dx$$

$$\cos^2 x + \sin^2 x = 1 \Rightarrow \cos^2 x = 1-\sin^2 x$$

$$\int (1-\sin^2 x)(3-\sin x)\cos x\,dx$$

$$\int (3-\sin x)\cos x\,dx - 3\int \sin^2 x \cos x\,dx + \int \sin^3 x \cos x\,dx$$

$$\underbrace{\boxed{3\int \cos x\,dx}}_{I_1} - \underbrace{\boxed{\int \sin x \cos x\,dx}}_{I_2} - 3\underbrace{\boxed{\int \sin^2 x \cos x\,dx}}_{I_3} + \underbrace{\boxed{\int \sin^3 x \cos x\,dx}}_{I_4}$$

$I_1 = -\sin x + C$

en I_2 $u = \sin x$; $du = \cos x\,dx$, $I_2 = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin^2 x + C$

$I_3 = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}\sin^3 x + C$

$I_4 = \int u^3\,du = \frac{1}{4}u^4 + C = \frac{1}{4}\sin^4 x + C$

4) $$\int_{x=0}^{x=1} x^2\sqrt{1-x^2}\,dx$$

$$x = \sin\theta \ , \quad dx = \cos\theta\, d\theta$$

$$\sqrt{1-x^2} = |\cos\theta|$$

$$\int_{\theta=0}^{\theta=\pi/2} \sin^2\theta \cdot \cos^2\theta\, d\theta$$

$$\int_{\theta=0}^{\theta=\pi/2} \left(\sin\theta\cos\theta\right)^2 d\theta$$

$$\int_{\theta=0}^{\theta=\pi/2} \left[\tfrac{1}{2}\sin(2\theta)\right]^2 d\theta \qquad \alpha = 2\theta \quad d\theta = \tfrac{1}{2}\,d\alpha$$

$$\frac{1}{2}\int_0^{\pi} \sin^2\alpha\, d\alpha \qquad \int_0^{\pi} \sin\alpha\, d\alpha = \left[-\cos\alpha\right]_0^{\pi} = -(-1)$$

$$\cos(2\alpha) = \underbrace{\cos^2\alpha}_{1-\sin^2\alpha} - \sin^2\alpha \ ; \ 2\sin^2\alpha = 1 - \cos(2\alpha) \ , \ \sin^2\alpha = \frac{1}{2} - \frac{1}{2}\cos(2\alpha)$$

$$\frac{1}{2}\int_0^{\pi} \sin^2\alpha\, d\alpha = \frac{1}{4}\int_0^{\pi}\left(1 - \cos(2\alpha)\right) d\alpha = \frac{1}{4}\underbrace{\boxed{\int_0^{\pi} d\alpha}}_{I_1} - \frac{1}{4}\underbrace{\boxed{\int_0^{\pi}\cos(2\alpha)\,d\alpha}}_{I_2}$$

$$I_1 = \pi \qquad \boxed{\beta = 2\alpha \ ; \ d\alpha = \tfrac{1}{2}\,d\beta}$$

$$I_2 = \int_0^{\pi} \cos(2\alpha)\,d\alpha = \frac{1}{2}\int_0^{\pi/2} \cos\beta\, d\beta = \frac{1}{2}\left[\sin\beta\right]_0^{\pi/2} = \frac{1}{2}$$

5(1) $\int \frac{1}{(x^2+4)(x^2-9)}\,dx$

$\int \frac{1}{x^2+4\,(x+3)\,(x-3)}\,dx \longrightarrow A=-\frac{1}{30};\ B=\frac{1}{30};\ D=-\frac{1}{5}$

$A\int\frac{A}{(x+3)}\,dx + B\int\frac{1}{(x-3)} + D\int\frac{dx}{x^2+4} \leftarrow u=\frac{x}{2}$

$2\cdot du = dx$

$A\ln(x+3) + B\ln(x-3) + D\boxed{\int\frac{2\,du}{4u^2+4}}$

$\int\frac{2\,du}{4u^2+4} = \frac{2}{4}\int\frac{du}{u^2+1} = \frac{1}{2}\,tg^{-1}u + c = \frac{1}{2}\,arctg\,u$

5(2)

$$\frac{1}{(x^2+4)(x+3)(x-3)} = \frac{A}{x+3} + \frac{B}{(x-3)} + \frac{Cx+D}{x^2+4}$$

$$A(x-3)(x^2+4) + B(x+3)(x^2+4) + (Cx+D)(x^2-9)$$

$$A(x^3+4x-3x^2-12) + B(x^3+4x+3x^2+12) + (Cx^3-9Cx+Dx^2-9D = 1$$

$$Ax^3+4Ax-3Ax^2-12A+Bx^3+4Bx+3Bx^2+12B+Cx^3-9Cx+Dx^2-9D=1$$

$$\begin{cases} A+B+C=0 & A+B=-C \Rightarrow A+B=0 \\ -3A+3B+D=0 \Rightarrow & A+B=0 \Rightarrow A=-B \\ 4A+4B-9C=0 & 4(A+B) = +9C = -4C \Rightarrow \boxed{C=0} \\ -12A+12B-9D=1 & \end{cases}$$

$$\begin{cases} A+B=0 \\ 3(B-A)=-D \\ 12(B-A)=1+9D \end{cases} > -4D = -1-9D \Rightarrow 5D=-1 \Rightarrow \boxed{D=-1/5}$$

$$\begin{cases} A+B=0 \\ B-A=\frac{1}{15} \end{cases} \Rightarrow \begin{cases} 2B=1/15 \\ 2A=-1/15 \end{cases} \quad \boxed{\begin{matrix} B=\frac{1}{30} \\ A=-1/30 \end{matrix}}$$